PhyzGuide: Algebraic Kinematics

I. DEFINITIONS

ENGLISH	EQUATIONS
Δ (Delta) means "change in," so Δ*n* means "change in *n*." Given two values of the quantity *n*, Δ*n* is calculated by subtracting the **first** value from the **second**.	$\Delta n = n_2 - n_1$ $\Delta n = n - n_0$
Velocity is defined as the rate at which a particle's position changes with respect to time. Therefore the velocity of a particle is the change in its position divided by the amount of time that passed during the particle's change in position.	$v \equiv \Delta x/\Delta t$
Acceleration is defined as the rate at which a particle's velocity changes with time. Therefore the acceleration of a particle is the **change** in its velocity divided by the amount of time that passed during the particle's change in velocity.	$a \equiv \Delta v/\Delta t$

II. ALGEBRAIC MANIPULATIONS

A. UM: Uniform Motion (no acceleration)

$v = x/t$

For a motion that involves *no* acceleration, this equation is sufficient.

CAUTION > **DO NOT USE THIS EQUATION FOR ANY MOTION INVOLVING ACCELERATION.** If you do, you will promptly dry up and blow away, or perhaps turn into a pumpkin.

B. UAM: Uniform Accelerated Motion

These equations are valid when there **IS** acceleration, and even when there **ISN'T**.

EQUATION	"WHO CARES" QUANTITY				
$v = v_0 + at$	x				
$x = v_0 t + {}^1/_2 at^2$		v			
$x = {}^1/_2(v_0 + v)t$			a		
$v^2 = v_0^2 + 2ax$				t	
$x = vt - {}^1/_2 at^2$					v_0

THE "WHO CARES" QUANTITY tells you which equation to use. For example, suppose you know *a*, *x*, and v_0 but not *v* or *t*. If you are looking for *t*, then you don't care about *v*, so use the equation whose "WHO CARES" quantity is *v*. If, however, you need to find *v*, use the equation whose "WHO CARES" quantity is *t*.

The Small Print

The equations above are written as they are usually used. But they leave room for some confusion. The t's above represent Δt's—the time interval, and not a specific point in time. And the x's above represent Δx's. When one is involved in heavy kinematics work, most Δ's become "understood" and unwritten. An alternate way of writing some of the above equations is

$v = v_0 + a\Delta t$ $\Delta x = {}^1/_2(v_0 + v)\Delta t$ $\Delta x = v_0\Delta t + {}^1/_2 a(\Delta t)^2$ $v^2 = v_0^2 + 2a(\Delta x)$

PhyzExamples: 1D Algebraic Kinematics

1. A marathon runner runs 26 miles in 3 hrs. What is the runner's average speed in m/s?

$x = 26$ mi = 42 km = 42,000 m

$t = 3$ hr = 10,800 s

$v = x/t$

$v = 42{,}000$ m / 10,800 s

<u>$v = 3.9$ m/s</u>

2. A skier starting from rest accelerates in a straight line down a slope at 2 m/s^2. How fast is he or she moving after 7 s? Distance *x* = 26 mi = 42 km = 42,000 m

$v_0 = 0$ $a = 2$ m/s^2 $t = 7$ s $v = x/t$

$v = v_0 + at$ $[v_0 = 0]$

"starting from rest" means $v_0=0$

$v = (2$ m/s$^2)$ (7 s)

<u>$v = 14$ m/s</u>

3. At this point, our skier plows into a snow bank and comes to rest in 0.50 s. What was the acceleration involved?

$v_0 = 14$ m/s $v = 0$ $t = 0.5$ s

"comes to rest" means v = 0

$v = v_0 + at$

$a = (v - v_0)/t$

$a = (0 - 14$ m/s) / 0.50s

$a = -28$ m/s2

4. How far did the skier plow into the snow bank during the 0.50 s of deceleration?

$v_0 = 14$ m/s $v = 0$ m/s $t = 0.5$ s

$x = 1/2(v_0 + v)t$ $[v_0 = 0]$

$= 1/2$ (14 m/s) · 0.5 s

$x = 3.5$ m

5. A stone is dropped from a height of 5 m. How long will it be in the air?

$y = 5$ m $v_0 = 0$ m/s $v = ?$ $a = 10$ m/s2 $t = ?$

$y = v_0t + {}^1/_2at^2$

$v_0 = 0$

$y = {}^1/_2at^2$

$t = \sqrt{(2y/a)}$

$t = \sqrt{(2 \cdot 5 \text{ m} / 10 \text{ m/s}^2)}$

$t = 1.0$ s

Use this equation because the WHO CARES quantity is v (we don't know v, and we don't need to know v)

6. A stone is dropped from a height of 5.0 m. What is its speed upon impact?

$y = 5$ m $v_0 = 0$ $v = ?$ $a = 10$ m/s^2 $t = ?$

$v^2 = v_0^2 + 2ay$

$v^2 = 2ay$

$v = \sqrt{(2ay)}$

$v = \sqrt{(2 \cdot 10\text{m/s} \cdot 5.0\text{m})}$

$v = 10$m/s

Use this equation because the WHO CARES quantity is t (we don't know t, and we don't need to know t)

7. If a car goes from rest to 20 m/s in 5 s, how far did the car go during this acceleration?

$x = ?$ $v_0 = 0$ $v = 20$ m/s $a = ?$ $t = 5$ s

$x = {}^1/_2(v_0 + v)t$ $[v_0 = 0]$

$x = {}^1/_2$ (20 m/s) · 5 s

$x = 50$ m

8. What was the acceleration during those 5 s?

$v = v_0 + at$

$a = (v - v_0)/t$ $[v_0 = 0]$

$a = (20$ m/s) / 5 s

<u>$a = 4$ m/s^2</u>