PhyzGuide: Newton’s Second Law

force and acceleration

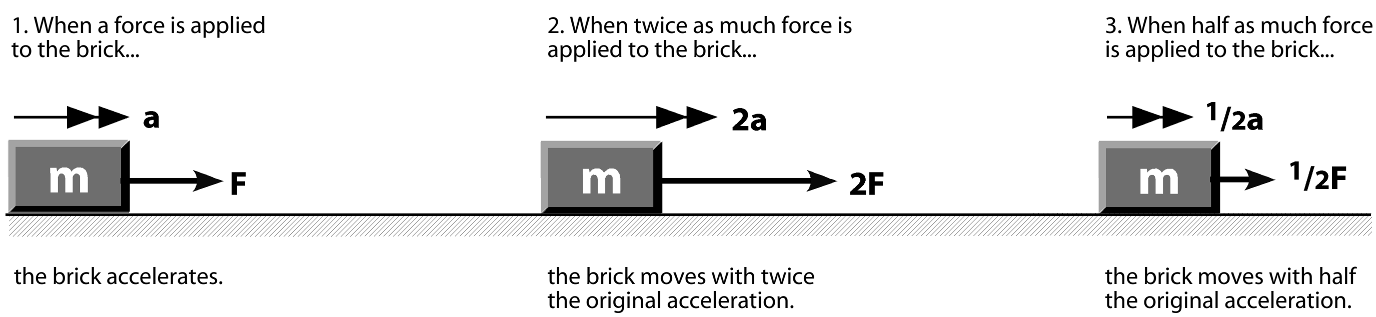
BECAUSE YOU CAN ONLY HAVE SO MUCH FUN WITH NEWTON I

A theory of dynamics must consistently explain three types of motion: **rest, uniform motion, and uniform acceleration**. Newton’s first law explains two of the three: rest and uniform motion. In both cases the net force on an object is zero—no unbalanced external force is acting.

So what happens when there **is** an unbalanced external force acting? Newton’s first law rules out rest and uniform motion. The only type of motion left is acceleration. The **converse** of Newton’s first law might be stated:

If an object is acted on by an unbalanced external force, it will accelerate.

OK, so it will accelerate. But at what rate: how fast will its speed be changing? Let’s go back to our brick on the table and accelerate it to discover how acceleration is related to other quantities. This time, however, we’re going to make the surface between the brick and table frictionless (it *is* possible to reduce friction so much that it becomes negligible).



This is an important finding: the acceleration of an object depends directly on the magnitude of the unbalanced external force. We have found a **direct proportionality:**

$$\boxed{a \propto F}$$

In English: The acceleration of a body is directly proportional to the net force acting on the body.

By knowing this direct proportion, we can deduce the acceleration of a body by knowing the net force acting on the body.

if	F	will cause	a	then
	$2F$	will cause	$2a$	
	$\frac{1}{2}F$	will cause	$\frac{1}{2}a$	
	$3.7F$	will cause	$3.7a$	
	$x\,F$	will cause	$x\,a$	

ah, but there is more to this mystery...

THE PLOT THICKENS…

So far, we've nailed down the relationship between the acceleration of a body and the net force acting on it. But there is more to the story.

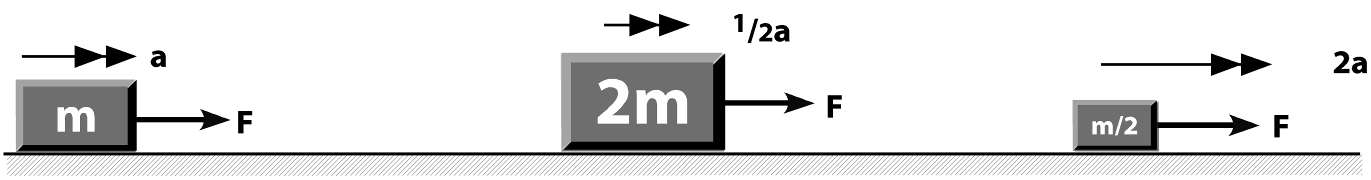
How would the acceleration produced by 1 unit of force be different if we applied it to a brick of twice the mass as the original? There are three choices here: The acceleration will be 1. **greater** than, 2. **less** than, 3. **same** as before.

If you think about it for a while (a nanosecond or so), or start pushing some objects around, you will settle on answer 2, less acceleration. Consider these three cases (again, bricks on our frictionless surface):

1. When a force is applied to the brick…

2. When a force is applied to a brick with twice the original mass…

3. If we apply the same force to a brick with half the original mass…



the brick moves with a certain acceleration.

the brick moves with half the original acceleration.

the brick moves with twice the original acceleration.

How very striking! As mass increases, acceleration decreases; as mass decreases, acceleration increases! It seems we have hit upon what mathematicians call an **inverse proportionality**:

$$\boxed{a \propto 1/m}$$

Or, in English: The acceleration of a body acted upon by a force is inversely proportional to the mass of the body.

By knowing this inverse proportion, we can deduce the acceleration of various bodies acted on by the same net force by knowing the masses of the bodies.

if	m	experiences	a	then
	$2\ m$	experiences	$a/2$	
	$^1/_2\ m$	experiences	$2\ a$	
	$3.7\ m$	experiences	$a/3.7$	
	$x\ m$	experiences	a/x	

Newton II in Symbols

$$\boxed{\mathbf{a} \propto \sum\mathbf{F}/m \quad => \quad \sum\mathbf{F} = m\mathbf{a}}$$

In English

The acceleration of a body is
directly proportional to the net force acting on the body and
inversely proportional to the mass of the body.