PhyzGuide: Conservation of Momentum

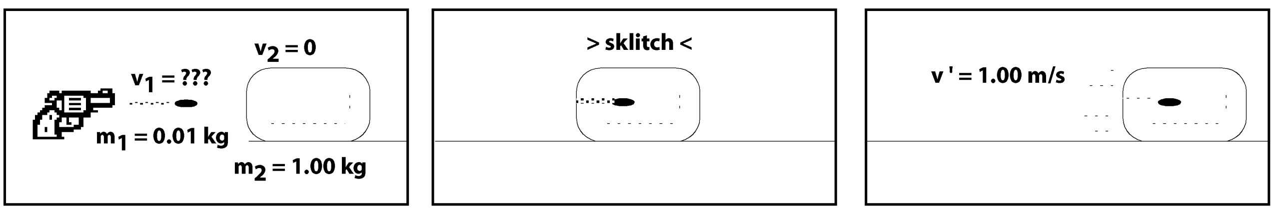
The real power of the momentum concept is a principle that physicists call "conservation of momentum." Conservation of momentum? What? Are we running out of it? Is this some kind of environmentalist thing? No, silly!

Conservation of momentum is an important "translation" of Newton's second and third laws. Simply stated, conservation of momentum means this: In the absence of external forces, the momentum of a system remains constant.

Hmmm.... *What's THAT supposed to mean?* It means that, for example, if two particles collide and aren't being acted on by any **outside** forces, the total momentum after the collision is equal to the total momentum before the collision. It doesn't even matter if they stick together or bounce off each other!

Thus in a collision, two particles may undergo vast changes in speed and direction of motion, but the momentum of the system remains unaltered. This is a powerful concept.

The numerical example below illustrates the power of this principle.



Consider a bullet $m_1 = 0.01$ kg fired at a block of ice $m_2 = 1.00$ kg. The block is initially at rest: $v_2 = 0$. The bullet enters the block and becomes lodged inside; the bullet and block move with a speed $v' = 1.00$ m/s. The mystery is "what's the initial speed of the bullet?"

The answer can be determined by knowing that the total momentum of the system remains constant before and after the collision.

Conservation of Momentum stated symbolically	$p = p'$
Momentum before the collision is p	$p = m_1v_1 + m_2v_2 = m_1v_1$ (since $v_2 = 0$)
Momentum after the collision is p'	$p' = (m_1 + m_2)v'$
Equating	$m_1v_1 = (m_1 + m_2)v'$
Solving	$v_1 = (m_1 + m_2)v' / m_1$
Putting in the numbers	$v_1 = (0.01 \text{ kg} + 1.00 \text{ kg}) \cdot 1.00 \text{ m/s} / 0.01 \text{ kg}$
	$v_1 = 101$ m/s

The bullet emerged from the gun with a muzzle speed of 101m/s.

COLLISIONS AND EXPLOSIONS: IMPORTANT DETAILS

1. Explosions

In a simple explosion, one object separates into two fragments. We generally consider the two masses m_1 and m_2 as initially combined. In all explosions, the speeds of m_1 and m_2 before the event are equal; that is, $v_1 = v_2$. We therefore use the symbol v to represent the initial speed. The masses of the fragments are not necessarily equal, so $m_1 \neq m_2$. The two objects will have different speeds after the event, so $v_1' \neq v_2'$.

SPECIAL CASES

a. Explosion from rest: $v = 0$

b. Equal fragments: $m_1 = m_2$ (=m, for simplicity of notation)

2. Inelastic Collisions

In an inelastic collision, two objects collide and stick together. Both objects have the same speed after the collision: $v_1' = v_2'$. We therefore use the symbol v' to represent the final speed of the compound object. The masses of the objects are not necessarily equal, so $m_1 \neq m_2$. The two objects can have different initial speeds, so $v_1 \neq v_2$.

SPECIAL CASES

a. Target at rest: $v_2 = 0$

b. Equal objects: $m_1 = m_2$ (= m, for simplicity of notation)

3. Elastic Collisions

In an elastic collision, two objects collide and bounce off each other. No simplifying assumptions to apply here. Initial speeds can be different, so $v_1 \neq v_2$. The masses of the objects are not necessarily equal, so $m_1 \neq m_2$. The two objects can have different final speeds, so $v_1' \neq v_2'$.

SPECIAL CASES

a. Equal objects: $m_1 = m_2$ (= m, for simplicity of notation)

b. HOPE. Although elastic collisions are generally difficult due to the number of unknowns and lack of simplifications, there is a set of "secret weapon" equations that result from conservation of momentum and conservation of energy considerations. These equations must be used with caution: they apply only to **h**ead-**o**n, **p**erfectly **e**lastic collisions when the target is at rest: $v_2 = 0$.

An object m_1 moving at v_1 undergoes a HOPE collision with an object m_2 initially at rest. What are the final speeds v_1' and v_2'?

$$v_1' = v_1 (m_1 - m_2) / (m_1 + m_2)$$

$$v_2' = 2m_1v_1 / (m_1 + m_2)$$