PhyzGuide: Momentum & Newton II

the INSIDE story

SCANDAL! THE PERVERSION OF NEWTON'S SECOND LAW

Isaac Newton first delineated what is now known as Newton's second law in 1686. During the interceding centuries, Newton's original version of this rule has been "rewritten." For purposes of education, textbooks and teachers have expressed this law as the familiar $\sum \mathbf{F} = m\mathbf{a}$. This is a useful statement, since it relates acceleration to force and mass. However, when Newton originally explained the relation between force and motion, he explained it in terms of what he called "quantity of motion."

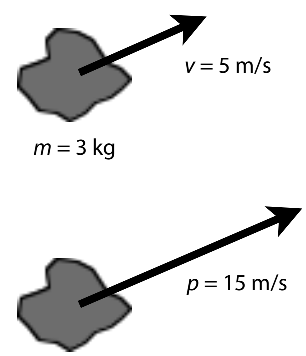
NEWTON II AS WE NOW USE IT: $\sum \mathbf{F}=m\mathbf{a}$ (the "perverted" version)

The net force acting on a body is equal to the product of the body's mass and its acceleration.

NEWTON II AS NEWTON ORIGINALLY REVEALED IT: $\sum \mathbf{F}=\Delta \mathbf{p}/\Delta t$

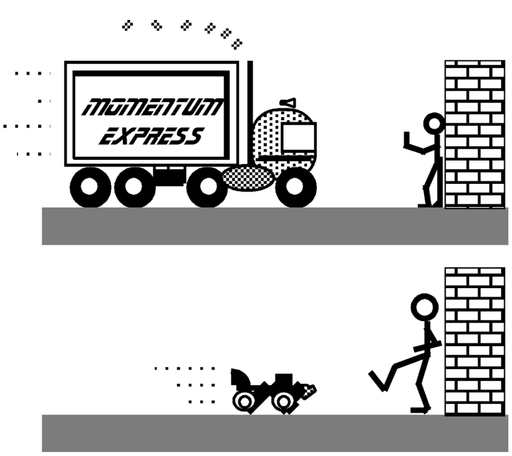
The net force acting on a body is equal to the rate at which the body's quantity of motion is changing. (p= "quantity of motion," the word "rate" implies division by time.)

Now that you have seen and used the "perverted" version of Newton's second law, you should see and know about the original version. It turns out to be quite useful.



Momentum is a vector whose magnitude is the product of mass and speed and whose direction is that of the velocity vector.

Newton's "quantity of motion" is what we now call "momentum." **Momentum** is the product of an object's mass and its velocity (velocity, not speed). In symbols, $\mathbf{p} = m\mathbf{v}$. The momentum of a body is a vector whose direction is the direction of the body's velocity. The magnitude of momentum is the product of mass and speed: $p = mv$. The SI unit of momentum is the kilogram-meter per second (kg·m/s) which is equivalent to the newton-second (N·s).



TRUCK STOP

Conceptually, think of momentum as "inertia in motion." Newton's first law explains that objects in motion want* to stay in motion. But just *how much* do moving objects wish to stay in motion? Does a 1 kg skate moving at 10 m/s "want" to stay in motion as much as a 10,000 kg truck moving at the same speed? To answer, think about which one would be harder to stop. Better yet, which one would you rather have approaching you?

*Please allow this anthropomorphism; and if you don't know what that means, you should look it up—before the test.

PhyzGuide: Algebraic Road Trip

from F = ma to F = Δp/Δt

CAN YOU GET THERE FROM HERE?

Let's look again at Newton's second law, and see how the "perverted" version relates to the original version. Certainly, they must say the same thing fundamentally. Or do they?

Is $\mathbf{F} = m\mathbf{a}$ an equivalent statement to $\mathbf{F} = \Delta\mathbf{p}/\Delta t$? Let's see how the algebra works out.

Perverted Newton II	$\mathbf{F} = m\mathbf{a}$
The definition of acceleration is	$\mathbf{a} = \Delta\mathbf{v}/\Delta t$
Therefore (substituting)	$\mathbf{F} = m\mathbf{a} = m(\Delta\mathbf{v}/\Delta t)$
Rearranging	$\mathbf{F} = (m\Delta\mathbf{v})/\Delta t$
The definition of momentum is	$\mathbf{p} = m\mathbf{v}$
Change in momentum is therefore	$\Delta\mathbf{p} = \Delta(m\mathbf{v}) = m\Delta\mathbf{v}$
So by substitution,	$\mathbf{F} = (m\Delta\mathbf{v})/\Delta t = \Delta\mathbf{p}/\Delta t$

Hey! That's Newton's original version!!! Incredible! I guess you ***can*** get there from here!!!

The point of all this algebraic hocus-pocus is that we now have a new way of analyzing motion and changes in motion. Note that we have *no* new laws—no new rules of the universe, we simply have a new way of stating and applying Newton's second law.

One advantage of using momentum is that it doesn't involve knowing acceleration. We will see the most important advantage of considering momentum later on in this unit.

ZEN SPACE