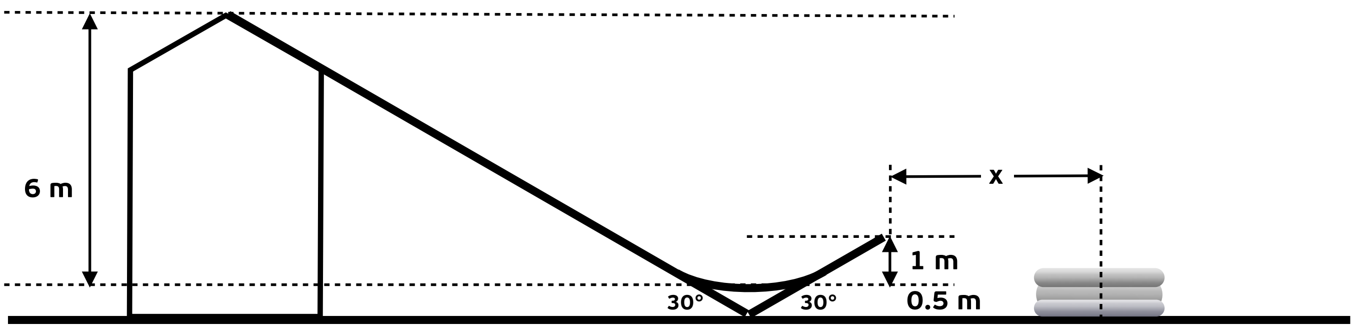
Felix

The Ultimate Backyard Water Slide

A backyard water slide enthusiast constructed a ramp from a portion of his home's roof to ground level. As is the case with Olympic ski jumps, the ramp then turns upward to launch whatever came down the ramp. The high end of the ramp is 6 meters above its lowest point, and the jump portion rises 1 meter above that low point. The ramp is inclined at 30° on both sides. A small pool is placed so as to catch the water slide rider, as shown in the illustration.



To establish the correct position the pool, a 7-kg bowling ball is rolled down the ramp (while it is still dry). and onto the lawn. The pool is then centered on the indentation made by the ball.

1. What will be the launch speed v_0 of the bowling ball when it reaches the end of the ramp?
a. Describe the strategy you will use to make this calculation. What is the story of this process? What is the headline (i.e., Kinematics, Conservation of Momentum, etc.) and what are the details (i.e., uniform gravitational acceleration of a body in free fall, the sum of the momenta of the bullet and target is equal to the momentum of the compound body, etc.)?

Potential energy at the top turns into kinetic energy at the bottom.
Kinetic energy is linear and rotational.
Effective height is 5 meters (not 6).

b. Calculate v_0.

$KE_{LIN} + KE_{ROT} = PE_5$

$KE_{LIN} = PE_5 - KE_{ROT}$

$1/2\ mv_0^2 = mgh - 1/2\ I\omega^2$

$1/2\ mv_0^2 = mgh - 1/2\ (2/5)\ mR^2\ (v_0/R)^2$

$1/2\ mv_0^2 + 1/2\ (2/5)\ mv_0^2 = mgh$

$v_0^2 + (2/5)\ v_0^2 = 2gh$

$v_0 = \sqrt{(2gh/(1 + 2/5))}$

$v_0 = \sqrt{(2\ 9.8m/s^2\ 5m\ /(1+0.4))} = 8.4\ m/s$

2. The distance, x, in the diagram is where the bowling ball lands. Calculate x.
a. Make an organized list of known relevant quantities.

$x = ?$

$v_x = 8.4 \text{m/s} \cos 30° = 7.3 \text{ m/s}$

$t = ?$

$y = -1.5 \text{ m}$

$v_{y0} = 8.4 \text{m/s} \sin 30° = 4.2 \text{ m/s}$

$v_y = ?$

$a = -g = -9.8 \text{ m/s}^2$

b. Calculate the distance x.

$x = v_x t$

t:

$y = v_{y0}t + 1/2at^2$

$(1/2)at^2 + v_{y0}t - y = 0$

$$t = \frac{-v_{y0} \pm \sqrt{[(v_{y0})^2 - 4(1/2)a(-y)]}}{2(1/2)a}$$

$$t = \frac{-4.2 \pm \sqrt{[(4.2)^2 - 4(1/2)-9.8(--1.5)]}}{2(1/2)(-9.8)}$$

$t = 1.13 \text{ s}$

$x = 7.3 \text{ m/s} \cdot 1.13 \text{ s}$

$x = 8.2 \text{ m}$

With the pool now in place, the water slide enthusiast dons his rubbery wetsuit and the rubbery ramp is doused with water. This creates a very low friction (aquaplaning) situation. When the enthusiast goes for his inaugural ride, the resulting video was recorded. [Watch video now.]

3. The outcome was not as predicted. Without explaining why, describe the simple nature of the error (in terms Goldilocks might understand).

He went too far; overshot the landing point.

4. Why did the enthusiast follow a different trajectory than the bowling ball did?

None of his potential energy turned into rotational kinetic energy.

5. Calculate a more accurate launch speed for the enthusiast.

$KE_{LIN} = PE_5$

$1/2\ mv_0^2 = mgh$

$v_0 = \sqrt{(2gh)}$

$v_0 = \sqrt{(2\ 9.8 m/s^2\ 5m)}$

$v_0 = 9.9\ m/s$

6. Calculate a more accurate landing point distance (x) for the enthusiast.

$x = ?$

$v_x = 9.9 m/s \cos 30° = 8.57\ m/s$

$t = ?$

$y = -1.5\ m$

$v_{y0} = v_0 \sin 30° = 4.95\ m/s$

$v_y = ?$

$a = -g = -9.8\ m/s^2$

$x = v_x\ t$

t:

$y = v_{y0}t + 1/2at^2$

$(1/2)at^2 + v_{y0}t - y = 0$

$$t = \frac{-v_{y0} \pm \sqrt{[(v_{y0})^2 - 4(1/2)a(-y)]}}{2(1/2)a}$$

$$t = \frac{-4.95 \pm \sqrt{[(4.95)^2 - 4(1/2)-9.8(--1.5)]}}{2(1/2)(-9.8)}$$

$t = 1.25\ s$

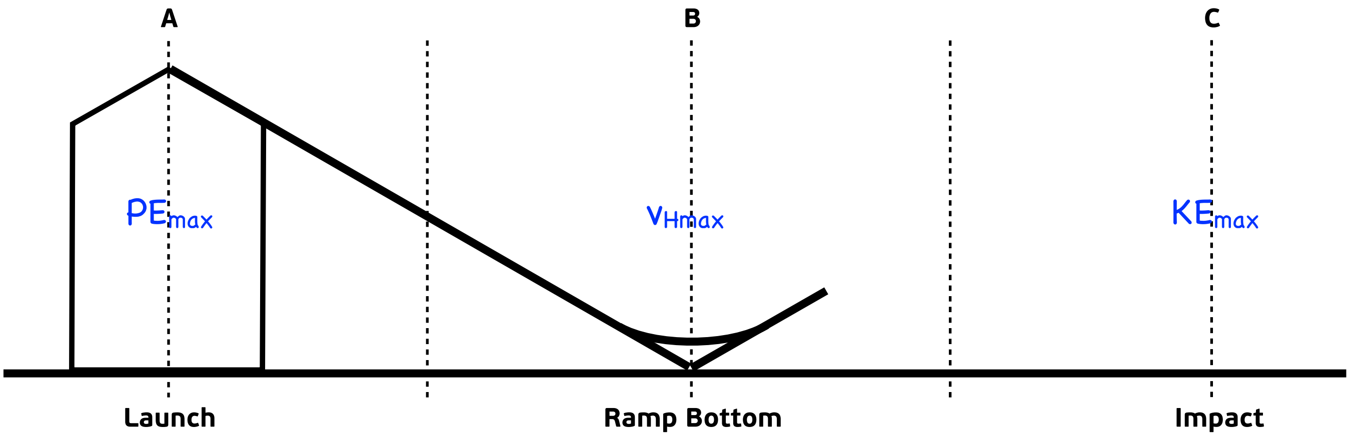
$x = 8.57\ m/s \cdot 1.25\ s$

$x = 10.72\ m$

7. Speculate: What would have happened if the pool had been placed at this newly-calculated distance? (Could it have resulted in a different injury to the enthusiast?)

Leg/ankle jam if feet hit pool wall while body is moving forward

Consider the water slide shown below. An enthusiast slides down a frictionless slide beginning at rest from the rooftop at A. He moves past the bottom of the slide at B and eventually becomes a projectile which impacts the ground at C. The horizontal distance AB is equal to the horizontal distance BC.



8. a. Draw the trajectory of the enthusiast from launch to impact.
b. Is the apex of the enthusiast's trajectory closer to ___B or ___C or ___same for both?
c. Does the enthusiast spend more time passing from _√_A to B or ___B to C or ___same for both? Briefly justify your choice.

d. Label the following points or segments along the trajectory.
i. maximum potential energy
ii. maximum kinetic energy
iii. maximum velocity in the horizontal direction

e. On the graph below, sketch a plot of the enthusiast's kinetic energy with a solid line and his potential energy with a dashed line, from launch to impact.