PhyzSpringboard: Bernoulli Equation

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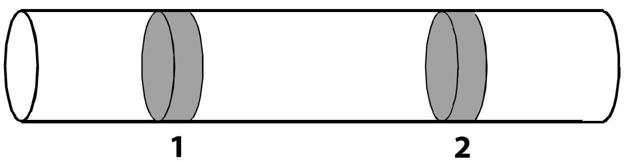
The Bernoulli equation is a consequence of conservation of energy as it applies to an ideal fluid moving in an ideal way. That is, the fluid has no viscosity and moves without turbulence.

Recall that the principle of conservation of momentum identifies a quantity that remains constant throughout a collision or explosion when no external forces act. And the principle of conservation of energy identifies a quantity that remains unchanged throughout a process such as a ball being thrown upward and falling back down.

The Bernoulli equation identifies a quantity that remains constant regardless of changes in elevation or speed of the fluid flow.

EASY BREEZY

1. Consider the simple flow of fluid through a pipe. The flow does rise or fall. Nor does it pass into a constriction or expansion.



a. How does the pressure at point 1 compare to the pressure at point 2?

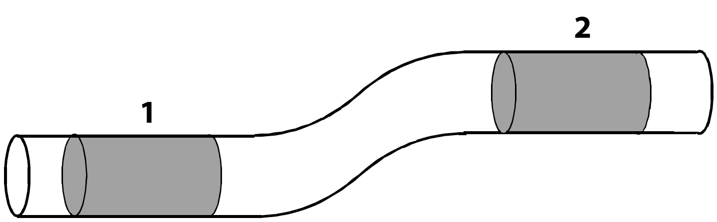
It is equal.

b. Write a mathematical expression, solving for P_1.

$P_1 = P_2$

UPHILL CLIMB

2. Consider a flow of fluid that climbs a rise in a pipe.



a. How does the pressure at point 1 compare to the pressure at point 2? Explain.

It is greater. Pressure increases with depth.

b. What does the fluid gain by climbing the rise?

Potential energy.

c. How much does it gain and where does it come from?

$PE = mg \quad h$ = Work done by surrounding fluid

d. Write expressions for the work done on the fluid as it climbs the rise

i. in terms of pressures and volume.

$W = \quad (PV) = P_1V - P_2V$

ii. in terms of the mass of the fluid, the height of the rise, and the acceleration due to gravity.

$W = mg \quad h$

e. i. Set the expressions equal to one another and solve for the difference in pressure.

$P_1V - P_2V = mg \quad h$

$P_1 - P_2 = mg \quad h/V$

ii. Simplify by recalling the definition of density.

$\rho = m/V$

$P_1 - P_2 = \rho g \quad h$

iii. Solve for P_1.

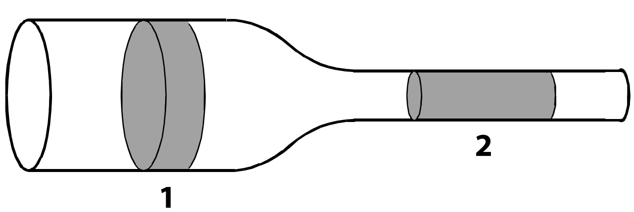
$P_1 = P_2 + \rho g \quad h$

f. Is this consistent with the variation of pressure with depth found in a static fluid?

Yes!

VENTURI EFFECT

3. Consider the flow of a fluid that passes into a constriction.



a. How does the pressure at point 1 compare to the pressure at point 2?

b. What does the fluid gain when it passes into the constriction?

Kinetic energy.

c. How much does it gain and where does it come from?

$KE = 1/2m(v_2^2 - v_1^2)$ = Work done by surrounding fluid

d. Write expressions for the work done on the fluid as it passes into the constriction
i. in terms of pressures and volume.

$W = \quad (PV) = P_1V - P_2V$

ii. in terms of the mass of the fluid, and the difference in fluid speed.

$W = 1/2m(v_2^2 - v_1^2)$

e. i. Set the expressions equal to one another and solve for the difference in pressure.

$P_1V - P_2V = 1/2m(v_2^2 - v_1^2)$
$P_1 - P_2 = 1/2m(v_2^2 - v_1^2)/V$

ii. Simplify by recalling the definition of density.

$\rho = m/V$
$P_1 - P_2 = 1/2\rho(v_2^2 - v_1^2)$

iii. Solve for P_1.

$P_1 = P_2 + 1/2\rho(v_2^2 - v_1^2)$

ALL TOGETHER NOW

4. Consider the flow of a fluid that climbs a rise into a constriction. **Your turn to draw!**

a. Use what you learned in parts 2 and 3 to write a mathematical expression for the pressure at point 1

$P_1 = P_2 + \rho g \quad h + 1/2\rho(v_2^2 - v_1^2)$

b. Remove the -notation and write the general form of this expression.

$P_1 = P_2 + \rho g(h_2 - h_1) + 1/2\rho(v_2^2 - v_1^2)$
$P_1 + \rho g h_1 + 1/2\rho v_1^2 = P_2 + \rho g h_2 + 1/2\rho v_2^2$