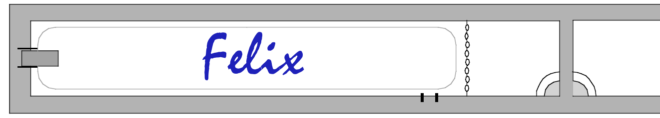
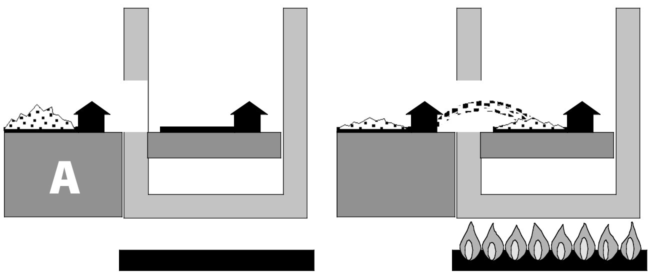
PhyzSpringboard: PV Cycles



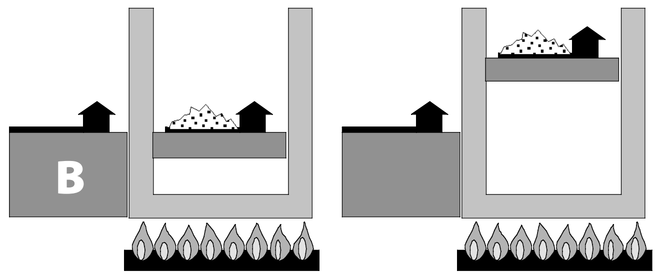
Heat engines are devices that turn thermal energy into mechanical energy. They use a series of thermodynamic processes that can be repeated. Such a series is called a **cycle.**

BASIC CYCLE: MY GRAIN ELEVATOR

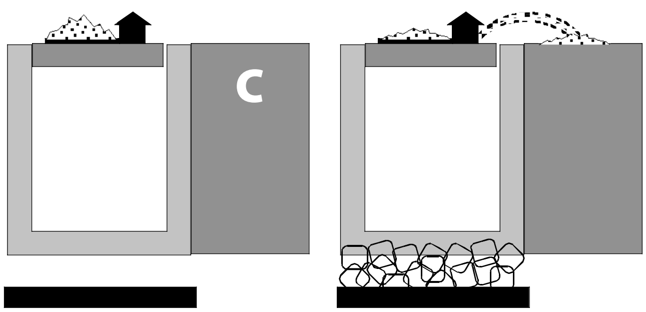
1. Consider a giant cylinder of gas capped by a piston that can move up or down in the cylinder without friction. Grain at the low elevation waits near the pitch house. Another pitch house rests on a piston in the cylinder of gas. The gas has an initial volume 20 m^3, pressure 50 N/m^2 and temperature 30 K.



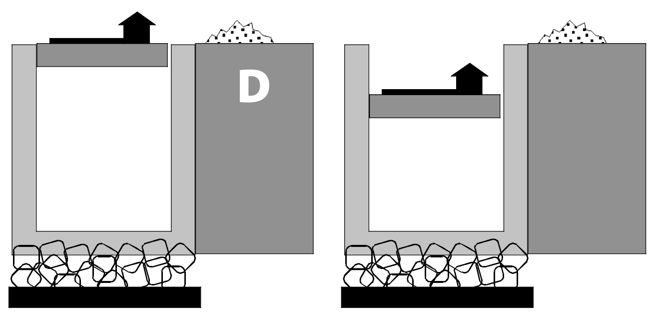
AB. As grain is pitched onto the elevator, heat is added to the gas. The elevator does not move up, but the pressure quadruples.



BC. Once the grain is fully loaded, the elevator door is shut. Heat added to the gas now causes the piston to rise. During this process, the volume of the gas triples.



CD. Once the piston reaches the high elevation, the heat is turned off. As grain is pitched off the elevator, the gas is cooled. During this process, the pressure is reduced to its original value.



DA. Once the grain is offloaded, continued cooling of the gas causes the piston to fall. During this process, the volume of the gas returns to its original value. This takes the gas to its original state.

a. What is the volume, pressure, and temperature at point A?

$V = 20\ m^3$, $P = 50\ N/m^2$, $T = 30\ K$

b. What is the volume, pressure, and temperature at point B?

$V = 20\ m^3$, $P = 200\ N/m^2$, $T = 120\ K$

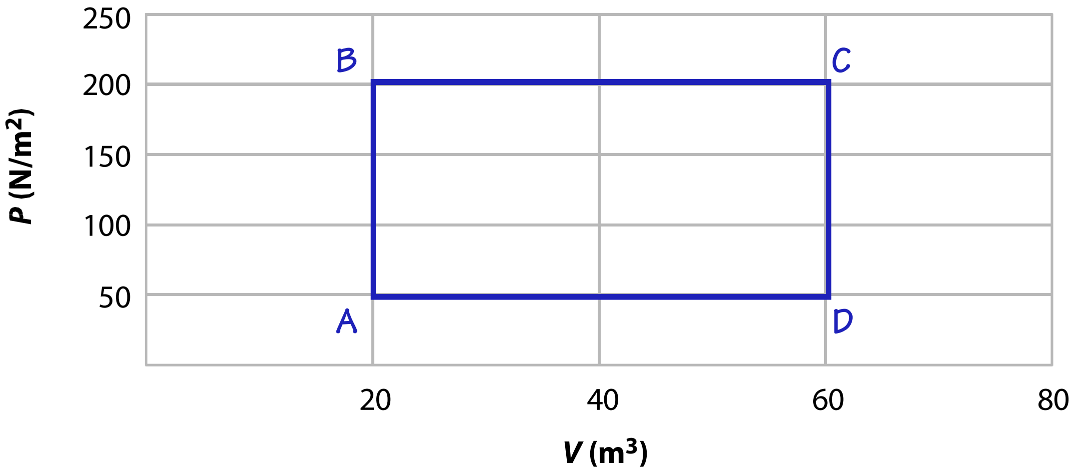
c. What is the volume, pressure, and temperature at point C?

$V = 60\ m^3$, $P = 200\ N/m^2$, $T = 360\ K$

d. What is the volume, pressure, and temperature at point D?

$V = 60\ m^3$, $P = 50\ N/m^2$, $T = 90\ K$

e. Plot the cycle on the pressure vs. volume graph that follows.



f. How much work was done **on** the gas during each process?

i. AB $W = -P\ \ V$: $V = 0$ so $W = 0$

ii. BC $W = -P\ \ V = -200\ N/m^2 \cdot (40\ m^3) = -8000\ J$

iii. CD $W = -P\ \ V$: $V = 0$ so $W = 0$

iv. DA $W = -P\ \ V = -50\ N/m^2 \cdot (-40\ m^3) = +2000\ J$

v. What was the net work done **on** the gas? (This is the work in, W_{in}.)

$W = -8000\ J + (2000\ J) = -6000\ J$

g. The molar specific heat capacities of the gas are $C_v = (3/2)R$ and $C_p = (5/2)R$. How much heat was added or removed in each process? (Hint: $PV = nRT$.)

i. AB $Q = nC_v\ \ T = n(3/2)R \cdot (P_BV/nR - P_AV/nR)$

$= (3/2)(P_B - P_A)(V_{AB})$

$= (3/2)(200\ N/m^2 - 50\ N/m^2)(20\ m^3) = 4500\ J$

ii. BC $Q = nC_p\ \ T = n(5/2)R \cdot (PV_C/nR - PV_B/nR)$

$= (5/2)(V_C - V_B)(P_{BC})$

$= (5/2)(60\ m^2 - 20\ N/m^2)(200\ N/m^2) = 20{,}000\ J$

iii. CD $Q = nC_v\ \ T = n(3/2)R \cdot (P_DV/nR - P_CV/nR)$

$= (3/2)(P_D - P_C)(V_{CD})$

$= (3/2)(50\ N/m^2 - 200\ N/m^2)(60\ m^3) = -13{,}500\ J$

iv. DA $Q = nC_p\ \ T = n(5/2)R \cdot (PV_C/nR - PV_B/nR)$

$= (5/2)(V_A - V_D)(P_{AD})$

$= (5/2)(20\ m^3 - 60\ m^3)(50\ N/m^2) = -5000\ J$

v. What was the total heat added from A to C? (This is the heat in, Q_{in}, and heat added, Q_H.)

Q_H = 4500 J + (20,000 J) = 24,500 J

vi. What was the total heat **ejected** from C to A? (Heat ejected, Q_C or Q_L, will be a **positive** value.)

Q_C = 13,500 J + (5000 J) = 18,500 J

vii. More heat was added than was ejected during the cycle. Account for the difference (explain what happened to the heat that was added but never ejected).

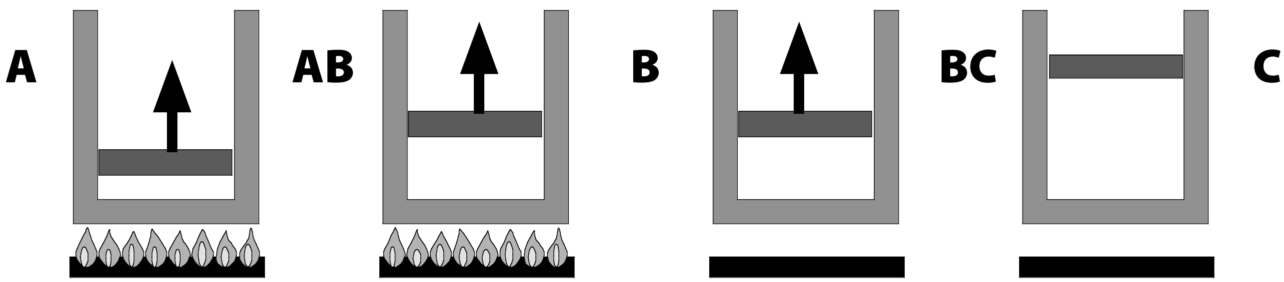
The heat that was added but never ejected became mechanical work done BY the engine during the cycle.

h. What was the efficiency of the cycle? Show two calculations.

$e = -W_{in}/Q_{in}$ = 6000J / 24,500J = 0.24 (24%)
$e = 1 - (Q_C/Q_H)$ = 1 − (18,500J / 24,500J) = 0.24 (24%)

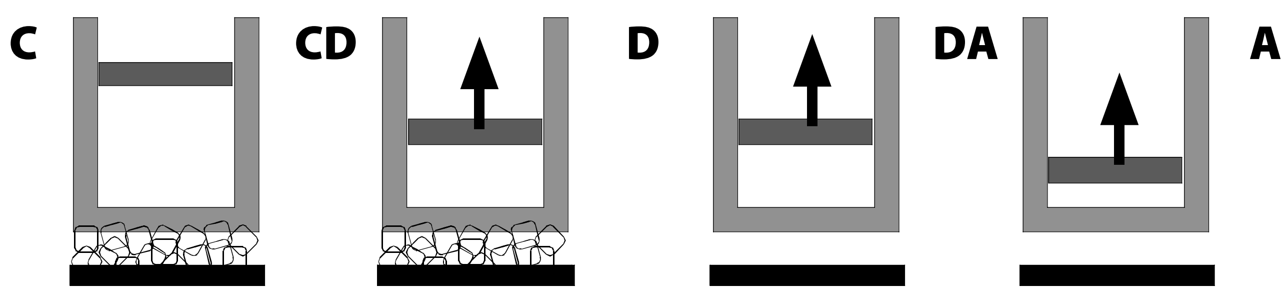
A MORE EFFICIENT CYCLE: THE CARNOT ENGINE

2. Consider a cylinder of gas going through the following cycle. The initial conditions are: P = 240 N/m
V = 20 m^3, and T = 360 K.



AB. As the story begins, the piston is rising. It had a "head start," in the sense that it was already moving up before we saw it. But 6545 J of heat are added nonetheless to keep it motivated. Since the gas is expanding upward faster than the added heat would force it to, the pressure is dropping. It drops to half its original value while the volume doubles. The temperature remains constant.

BC. The heat is turned off, but the piston continues to rise and the pressure continues to drop. The pressure falls to 1/30th its original value as the volume increases to 2.5 times its original value.

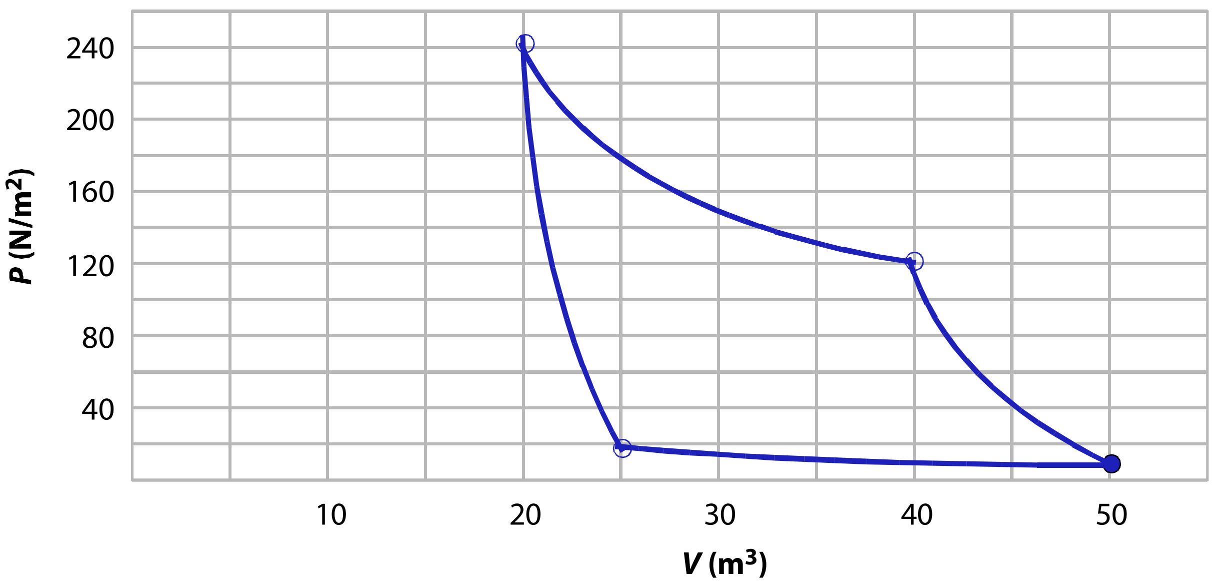


CD. Heat is removed. The gas contracts to half its previous volume. The pressure increases to double its previous value. The temperature remains constant.

DA. No heat is added or removed. The gas continues to contract and the pressure rises. The temperature increases. When all is done, the gas has returned to its original state.

a. Determine the volume, pressure, and temperature at points B, C, and D. Plot the cycle on the axes below. Identify points A-D and label the temperatures of the isotherms.

A	B	C	D
$P = 240\ N/m^2$	$P = 120\ N/m^2$	$P = 8\ N/m^2$	$P = 16\ N/m^2$
$V = 20\ m^3$	$V = 40\ m^3$	$V = 50\ m^3$	$V = 25\ m^3$
$T = 360\ K$	$T = 360\ K$	$T = 30\ K$	$T = 30\ K$



b. What is the Carnot efficiency of this engine?

$e = 1 - (T_C / T_H)$

$e = 1 - (30K / 360K)$

$e = 0.92 = 92\%$

c. How much work was done ON the gas from A to B?

$U = Q + W$

$W = U - Q$

$U = 0$ for isothermal processes

$W = -Q = -6545\ J$

d. How much heat is ejected from C to D?

$e = 1 - (T_C / T_H) = 1 - (Q_C / Q_H)$

$(T_C / T_H) = (Q_C / Q_H)$

$Q_C = Q_H (T_C / T_H)$

$Q_C = Q_H (T_C / T_H)$

$Q_C = 6545J (30K / 360K)$

$Q_C = 545J$

e. What was the net work done BY the engine during one cycle?

For one cycle:

$Q = U + W$

$U = 0$ (final T = initial T)

$W = Q$

$W = Q$

$W = 6545J - 545J$

$W = 6000J$