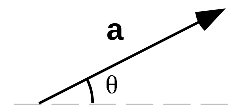
PhyzGuide: Resolving & Composing

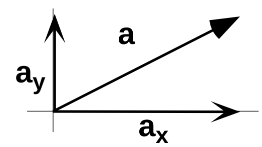
HOW ME 'N' BILLY-BOB DONE BUSTED UP SOME VECTORS REAL GOOD-LIKE!

Isn't this vector math just the greatest? So far, we've learned how to multiply vectors by scalars, add vectors, and subtract vectors.

But what if you were given some vectors to add, and they were expressed in **polar components** instead of the usual **rectangular components**? There is no direct method of adding polar components (as there is for rectangular components). To add these vectors, we must *first* convert magnitude and direction into *x*- and *y*-components and *then* add them. Here's how it's done.



POLAR: $\mathbf{a} = (a;\ \theta)$



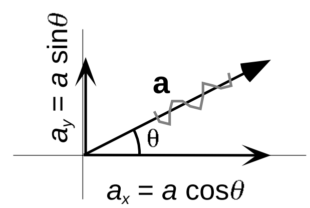
RECTANGULAR:
$\mathbf{a} = (a_x,\ a_y)$

Recall:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{a_y}{a}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{a_x}{a}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{a_y}{a_x}$$

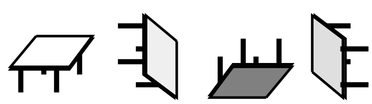


The vector **a** is "wiggled out" to show we've resolved it into components.

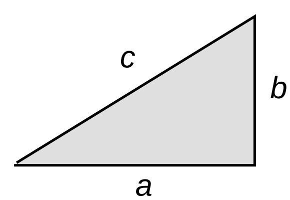
Some smart cross-multiplication yields these important and memorable results—the components we set out to find:

$$a_x = a\cos\theta \qquad \text{and} \qquad a_y = a\sin\theta.$$

TURNING THE TABLES



We can get the rectangular components of a vector from its magnitude and direction, but can we calculate magnitude and direction of a vector from its rectangular components? Of course we can—*we're physicists! (Or at least physics types.)*



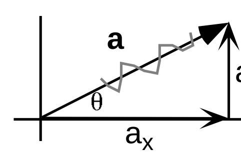
$a^2 + b^2 = c^2$

Just dust off your Pythagorean Theorem and get it dirty! According to Pythagoras, the relationship between the lengths of the legs of any right triangle is

$$a^2 + b^2 = c^2.$$

We can use this on the right triangle formed by a, a_x, and a_y. If we know the components a_x and a_y, we *can* solve for a

$$a^2 = a_x^2 + a_y^2 \quad \rightarrow \quad a = \sqrt{(a_x^2 + a_y^2)}.$$



$a^2 = a_x^2 + a_y^2$

"Cake," you say, "but how do you propose we squeeze a θ out of a_x and a_y?" ¡No Problema! Look at our definition of tangent above. A simple rearrangement yields:

$\theta = \text{Tan}^{-1}\ (a_y/a_x)$. [Tan^{-1} is also known as INVerse tangent or ARCtangent.]