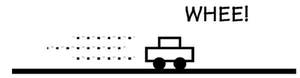
PhyzGuide: Projectiles

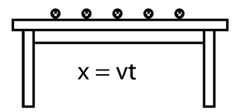
Motion in 2 Dimensions



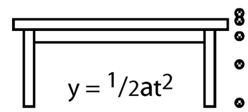
Now that we are masters of one-dimensional kinematics, it is time to move into the world of two dimensions. With 2-D kinematics, we can study **projectile motion.** Whereas in 1-D we could only study particles confined to moving in a straight line, we can now study objects that move along in an arc: baseballs, snowballs, tomatoes, eggs, spit-wads, etc.

At first this may seem like an intimidating task: How can one keep track of such a complicated motion? The answer is a good news/bad news combination. The good news is that 2-D kinematics is incredibly simple: you don't need to learn *any* new equations. The bad news is that you may refuse to accept the good news above and insist on making 2-D kinematics more difficult than it is.

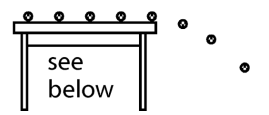
Here's the deal: Suppose a marble rolls horizontally along a table. It will continue to roll along at a constant speed (neglecting friction, as always). We already know how to describe *that* motion. It's a simple 1-D kinematics problem. The distance traveled by the marble is $x = vt$.



Suppose you were to drop the marble from the edge of the table. It would, of course, accelerate due to gravity. We already know how to deal with *this* 1-D motion as well. The distance traveled by the marble is $y = {}^{1}/_{2}at^{2}$.



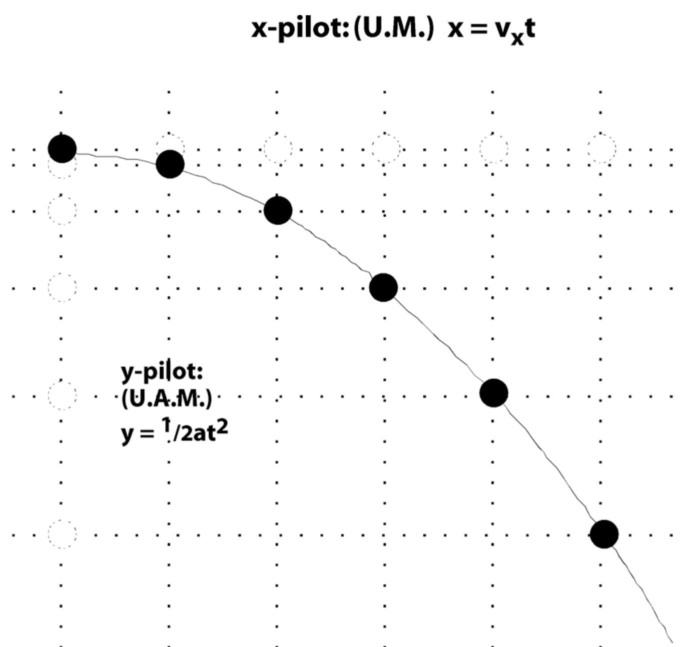
Now suppose you roll a marble so that it rolls at a constant horizontal velocity and then rolls off the edge of the table. It becomes a projectile. It undergoes motions in two directions at once.



In the x-direction, the marble continues at its original v_x. Remember: there is nothing in the universe acting to accelerate the marble in the *x*-direction. The equation for motion in the *x*-direction remains $x = v_x t$.

In the *y*-direction, the marble is accelerating as if it had been dropped from rest. The equation for motion in the *y*-direction is $y = {}^{1}/_{2}at^{2}$.

A 2-D kinematics puzzle, then, is nothing more than two 1-D kinematics puzzles happening at the same time.



The trick to solving such puzzles is to think of the projectile as a "flying machine" with two pilots: an *x*-pilot, and a *y*-pilot.

The *x*-pilot is programmed only for motion in accordance with $x = v_x t$.

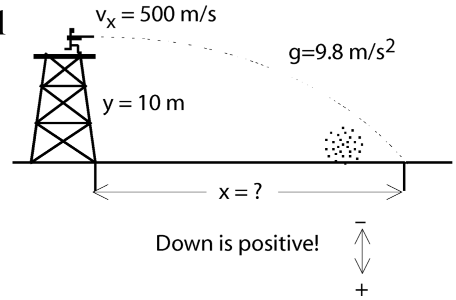
The *y*-pilot is programmed for motion following the general equation $y = v_{y0}t + {}^{1}/_{2}at^{2}$.

Suppose you were asked, "How much time will pass between the point when the marble leaves the edge of the table and when it hits the floor?"

To solve this, ask yourself, "Which pilot will be the first to know when the motion stops?" In this case, it is the *y*-pilot, so you can determine *t* from the *y*-pilot's equation. You can *not* determine *t* from the *x*-pilot's equation.

PhyzExample: Projectile Motion

A rifle is fired horizontally from a platform 10 m above level ground. The muzzle velocity (the speed at which the bullet emerges from the barrel of the gun) is 500 m/s. How far downrange will the bullet hit the ground?



Before doing anything else, we must decide "which way is up?" meaning: is the direction up positive or negative? Since our bullet is going DOWN and acceleration due to gravity is DOWN, I'm going to choose DOWN as positive. I could have chosen up as positive, but then y = –10 m and a = – 9.8 m/s², and I prefer positive numbers to negative numbers.

To solve the problem, first write down what you are GIVEN and what you can DEDUCE on a complete table of quantities.

x: UM	y: UAM
$x = ?$	$y = 10$ m
$v_x = 500$ m/s	$v_{y0} = 0$ m/s
	$v_y = ?$
	$a = 9.8$ m/s2

$t = ?$

THE FINE PRINT:

x: x is the unknown we're ultimately looking for.

vx: Velocity in the x-direction does not change since there is NO natural acceleration in the x-direction.

t: Time for the flight is not associated with x or y—it is the same for both; and we don't know it.

y: y is +10 m because we chose down to be positive.

vyo: Initial velocity in the y-direction is 0 m/s because the rifle is aimed HORIZONTALLY—no upward or downward component.

vy: We don't know how fast the bullet will be traveling in the y-direction when it hits the ground, so vy is unknown.

a: Acceleration in the y-direction is +9.8 m/s2 because we chose down to be positive.

The question asks us how far downrange the bullet will land. This is a question about x. Motion in the x-direction is uniform motion (no acceleration), and is therefore described by

$$x = v_x t.$$

This is the equation that will solve our problem. Unfortunately, we cannot use it yet. For although we ARE solving for x, and we DO know v_x, we do NOT know t—the time for the flight.

Question: which pilot will know first when the bullet hits the ground: the x-pilot or the y-pilot?

Answer: the y-pilot. It is the motion in the y-direction that limits the flight. The bullet is limited to travel only 10 m in the y-direction, whereas it could technically travel any distance in the x-direction.

So we can determine the time of flight by calculating how long it would take the bullet to fall 10 m.

THE TIME IT TAKES THE BULLET TO FALL 10 m IS THE TIME THE BULLET REMAINS IN THE AIR.

The general equation governing motion in the y-direction is:

$$y = v_{y0}t + {}^1\!/_2 at^2$$

Since $v_{y0} = 0$ m/s, the equation can be simplified to

$$y = {}^1\!/_2 at^2$$

To solve the x equation above, we need to know t. Solving the y equation for t gives

$$t = \sqrt{(2y/a)}$$

ALGEBRA:
$y = {}^1\!/_2 at2$
$2y = at2$
$2y/a = t2$
$t = \sqrt{(2y/a)}$

Since we were asked for x, not t, we DON'T plug in any numbers yet! Instead, we substitute the t we just found into the x equation, and only THEN plug in numbers.

$$x = v_x t$$

$$x = v_x\sqrt{(2y/a)}$$

$$x = 500 \text{ m/s} \sqrt{((2)(10 \text{ m})/(9.8 \text{ m/s}_2))}$$

$$x = 714 \text{ m}$$