THE RULES AND HOW TO FOLLOW THEM

FBD problems can look mighty ugly (if not downright impossible). However, an answer can always be found if you stay calm and follow this procedure.

0. Sketch the forces. Before you do anything else, sketch the forces in your picture of the problem. If you don't have a picture of the problem, make one. If the problem involves a pulley or an inclined plane, make a second sketch to "flatten" the problem.

1. Draw a diagram. Draw a **free-body diagram** (FBD) for each object of interest. If there's more than one object, handle each one separately. You draw an FBD by showing the object as a point (a dot) and extending arrows from the point indicating all external forces that act on the object. *Do not show forces that the object exerts on its surroundings.*

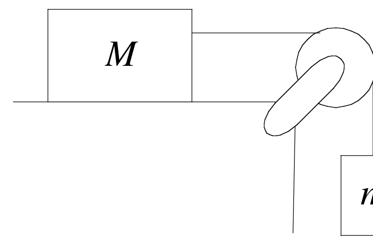
2. Label the forces. Make sure you draw **all** external forces acting on the particle(s). Check for the following: weight W, tension T, normal N, friction f, drag (air resistance) D, lift L.

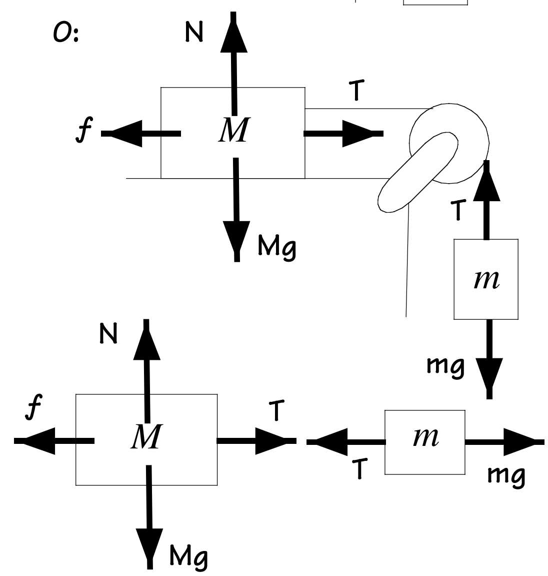
3. Show your coordinates. Choose the best (easiest) coordinate system. You don't have to stick to the standard axes if rotating those axes will make the problem easier to solve. There is no absolute coordinate system in the universe, so you are free to choose any system you like.
- UM or UAM: Choose the $+x$ and $+y$ direction. They must be perpendicular to each other. ***If acceleration is occurring, make the direction of acceleration one of the directions (+x or +y).***
- UCM: Choose $+r$ (radial) toward the center of the circle.

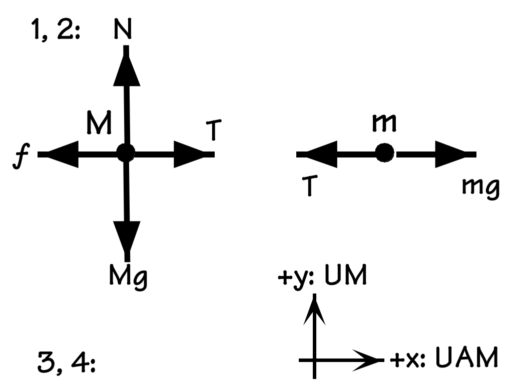
4. Indicate the type of motion the object is undergoing by writing near the diagram "UM" if the object is at rest or moving with constant velocity, "UAM" if the object is undergoing linear acceleration, or "UCM" if the object is in uniform circular motion. When the system is accelerating, it usually accelerates in one dimension while maintaining uniform motion in the other.

5. Resolve forces into *x* and *y* components. If you have any forces that are not acting directly along the x or y axes, "wiggle out" the force vector and draw the x and y components instead. Don't bother with the trigonometry of resolving the force vector into components yet.

Example problem:
A block with mass M = 3.6 kg is attached to a block with an unknown mass by a massless cord that passes over a massless, frictionless pulley as shown. If the coefficient of friction between the table and block is 0.42 and the system accelerates at 1.5 m/s^2, what is the mass of the hanging block?







—this problem is confined to x- and y-directions—

6. Apply Newton's second law to the system by first writing $\sum F = ma$. If the problem is two-dimensional, break it down into two one-dimensional problems by writing $\sum F_x = ma_x$ and $\sum F_y = ma_y$ separately.

6:

M: $F_x = ma_x$ $F_y = ma_y$

m: $F_x = ma_x$

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7. Substitute for "∑F." Underneath $\sum F$, rewrite the left-hand side of the equation by writing down a summation of the actual forces (tension, weight, etc.) acting on the particle. Consider all forces as magnitudes and indicate their direction by using + and – signs (+ if in the positive *x* or *y* direction, – if in the negative *x* or *y* direction).

7:

M: $F_x = ma_x$ $F_y = ma_y$

$T - f =$ $N - Mg =$

m: $F_x = ma_x$

$mg - T =$

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8. Substitute for "ma." Following the summation in step 7, rewrite the "*ma*" part of the Newton II equation to indicate the kinematic situation at hand:

- UM: If the object is in uniform motion (including at rest), then $a = 0$ and therefore $ma = 0$.
- UAM: If the object is undergoing linear acceleration, then consider the direction of acceleration (+/–) and the symbol you are using to represent the mass of the object (e.g. *m*, *M*, etc.) and replace *"ma"* with *ma, –ma, Ma,* or *–Ma*. If the object is undergoing free fall, replace *a* with *g*.
- UCM: If the particle is in uniform circular motion, then $a = v^2/r$ and therefore $ma = mv^2/r$.

8:

M: $F_x = ma_x$ $F_y = ma_y$

$T - f = Ma$ $N - Mg = 0$

m: $F_x = ma_x$

$mg - T = ma$

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9. Substitute for anything else. Make all other *appropriate* substitutions (such as $f = \mu N$, etc.) and trigonometrically resolve forces into components ($T_x = T\cos\theta$, etc.).

9:

M: $F_x = ma_x$ $F_y = ma_y$

$T - f = Ma$ $N - Mg = 0$

$T - \mu N = Ma$ $N = Mg$

m: $F_x = ma_x$

$mg - T = ma$

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10. Solve for the unknown quantity you've been asked to find. If you had a one-dimensional problem, you have one equation; if the problem was two-dimensional, you now have two equations. NOTE: you may need to delve into a kinematics problem to determine a force (by relating the force to acceleration via $F = ma$). REMEMBER TO ALWAYS SOLVE FOR THE LETTERS FIRST! Don't put numbers in until you have an equation for the unknown quantity in terms of known quantities. Putting numbers in too soon turns your paper into a dance floor for the red pen.

10:

$T - \mu N = Ma$ [*M in the x*]

$T = Ma + \mu N$

$mg - T = ma$ [*m in the x*]

$T = mg - ma$

$Ma + \mu N = mg - ma$ [*equating T's*]

$N = Mg$ [*M in the y*]

$Ma + \mu Mg = m(g - a)$

$m = M(a + \mu g) / (g - a)$

$m = \dfrac{3.6\ \text{kg}\ (1.5\ \text{m/s}^2 + 0.42 \cdot 9.8\ \text{m/s}^2)}{(9.8\ \text{m/s}^2 - 1.5\ \text{m/s}^2)}$

$m = 2.4\ \text{kg}$

The cool thing about these FBD problems (if there **is** a cool thing about them) is that they all work the same way. You have to know that (believe that) going into the problem. If you carry out the procedure above, you can get the correct answer. You won't get the right answer if you try to figure it out in your head or make a half-buttocksed attempt by following a couple of parts of the procedure and winging the rest. The right answer is achieved only by following the full procedure.

The only way to get good at solving these problems is through practice. They're a piece of cake when someone else is solving them, but someone else won't be solving them for you on the test. And you will need to know how to solve these problems for the test.