PhyzGuide: Terminal Speed for Poets

WHEN IT RAINS, IT POURS...

Consider the impact speed of raindrops falling from a cloud. If a drop starts in a cloud at a height h = 1200 m above the surface of the earth, how fast would it be going when it hit the ground (or a person, car, or building, etc., near the ground)? Let's haul out some kinematics problem-solving power and find out!

$x = 1200\ \text{m}$
$v_0 = 0$
$v = ?$
$a = 9.8\ \text{m/s}^2$
$t = ?$

$$v^2 = v_0^2 + 2ax$$
$$v = (2ax)$$
$$v = (2 \cdot 9.8\ \text{m/s}^2 \cdot 1200\ \text{m})$$

v = 153 m/s = 340 mph

Thankfully, raindrops do not hit us at 340 mph; serious damage would result if they did. And a hail storm with even the smallest hailstones moving at this speed would seem like machine-gun fire from on high!

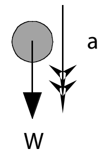
...OR DOES IT?

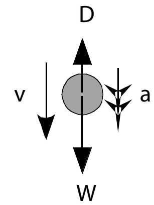
Umbrellas do not need to be made from titanium alloys or reinforced steel because raindrops and hailstones hit the ground with a speed much less than 340mph. This is due to a friction-like force that objects encounter when moving rapidly through a fluid. The force is called **drag**. Like kinetic friction, drag acts in the direction opposite to the velocity of an object in motion. The magnitude of the drag force acting on an object moving through a fluid depends on the cross-sectional area of the object, the shape of the object, the density of the fluid, and the speed of the object relative to the fluid. We write this relationship as $D = kv^2$, where k is the composite of area, shape, and fluid density, and v is speed. The drag acting on a raindrop as it falls through air keeps it from gaining the 340 mph speed it would in the absence of air. Drag prevents the raindrop from accelerating beyond a certain speed...

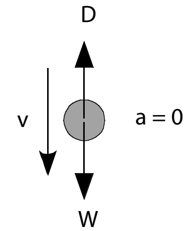
TERMINAL SPEED

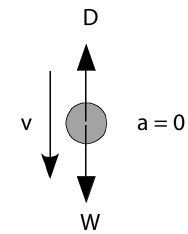
As an object moves faster and faster through a fluid, the drag force becomes greater and greater ($D \propto v^2$). A falling object accelerates due to the gravitational force ($W = mg$) exerted on it by the earth. As the object accelerates, however, its speed increases and the drag on it becomes greater and greater until it is equal to the weight of the object. At this point, the net force on the falling object is zero, so it no longer accelerates. Its speed now remains constant; it is traveling at its **terminal speed**. Terminal speed occurs when the weight force (down) is equaled by the drag force (up), in other words (or symbols), when $mg = kv^2$. But weight—I mean *wait*! If the downward weight force is equaled by the upward drag force, doesn't that mean the object must be at rest? In a word, NO! Remember, oh Aristotelian one, balanced forces occur when an object is at rest *or moving with constant velocity*. Our descending object falls with constant velocity when it reaches terminal speed.

AN OBJECT FALLS AND REACHES TERMINAL SPEED









1. The object begins its fall. It has no speed; the weight force causes a downward acceleration at 9.8 m/s^2.

2. As the object picks up speed, the upward drag force increases. This reduces the downward net force; the downward acceleration is reduced.

3. When the drag force matches the weight, the net force is zero and acceleration ceases. The object continues to fall at terminal speed.

AND THE RAINDROP?

How fast *is* a raindrop traveling when it hits the ground? (A typical raindrop has a mass $m = 1.5 \times 10^{-6}$ kg and an aerodynamic coefficient $k = 2.5 \times 10^{-6}$ kg/m.)

$$mg = kv^2$$
$$v^2 = mg/k$$
$$v = (mg/k)$$

$$v = ((1.5 \times 10^{-6}\ \text{kg} \cdot 9.8\ \text{m/s}^2) / (2.5 \times 10^{-6}\ \text{kg/m}))$$

v = 7.7 m/s = 17 mph

This is a much "kinder and gentler" speed and is far less damaging than the 340 mph calculated without drag. Modern scholars doubt that William Shakespeare ever made this calculation, but offer no other explanation for the bard's passage in *The Merchant of Venice* wherein Portia characterizes mercy by saying that

...it droppeth as the gentle rain from heaven, upon the place beneath...

Creative imagery or physical insight? You decide.

Some Terminal Speeds in Air

Object	*Terminal Speed (m/s)*	*mph*	*95% Distance* (m)*
16-lb shot	145	324	2500
Sky diver (typical)	60	134	430
Baseball	42	94	210
Tennis ball	31	69	115
Basketball	20	45	47
Ping-Pong Ball	9	20	10
Raindrop (r = 1.5mm)	7	16	6
Parachutist (typical)	5	11	3

* 95% Distance is the distance through which the body must fall from rest to reach 95% of its terminal speed.

Source: Adapted from P.J. Brancazio, *Sport Science*, Simon & Schuster, New York, 1984 in Halliday & Resnick's *Fundamentals of Physics, Third Edition,* John Wiley & Sons, 1988.

