PhyzGuide: Rotational Dynamics II

MOMENTUM

translational

Name Linear Momentum
Symbol p
Defn Momentum is "quantity of motion," and is defined as the product of an object's mass and velocity:

Eqn $p = mv$

Vector Direction of the vector **p** is the direction of the object's velocity.

rotational

Name Angular Momentum
Symbol L (also ℓ)
Defn Angular momentum is "quantity of rotation," and is defined as the product of an object's rotational inertia and angular velocity:

Eqn $L = I\omega$ (also, $L = mvr$)

Vector Direction of the vector **L** is the object's axis of rotation (via a right hand rule).

IMPULSE

translational

Name Linear Impulse
Symbol Δp
Defn Linear impulse is the change in linear momentum. It is brought about by an unbalanced external force acting on an object over an interval of time.

Eqn $\Delta p = F\Delta t = \Delta(mv)$

Vector Δ**p** is in the direction of **F.**

rotational

Name Angular Impulse
Symbol ΔL
Defn Angular impulse is the change in angular momentum. It is brought about by an unbalanced external torque acting on an object over an interval of time.

Eqn $\Delta L = \tau\Delta t = \Delta(I\omega)$

Vector Δ**L** is in the direction of τ.

CONSERVATION

translational

Conservation of linear momentum
If a system is isolated for linear motion (the sum of all external forces is zero), then its linear momentum is conserved (i. e., remains unchanged after an internal interaction).

$$p = p' \Rightarrow m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$$

Angular momentum is separate from linear momentum when considering conservation laws: **p** is conserved and **L** is conserved. Kinetic energy does not work this way, as we shall see...

rotational

Conservation of angular momentum
If a system is isolated for rotational motion (the sum of all external torques is zero), then its angular momentum is conserved (i. e., remains unchanged after an internal interaction).

$$L = L' \Rightarrow I\omega = I'\omega'$$

Angular momentum is conserved as a spinning skater pulls in his/her arms. The skater undergoes an **increase** in angular speed while undergoing a simultaneous **decrease** in rotational inertia.

PhyzGuide: Rotational Dynamics III

translational — WORK — rotational

Name Work
Symbol W
Defn Work occurs in the linear sense when a force acts to translate an object through a distance. It can increase or decrease the total energy in a particular system.

Eqn $W = \mathbf{F} \cdot \mathbf{d} = Fd\cos\phi$

Name Work
Symbol W
Defn Work occurs in the rotational sense when a torque acts to rotate an object through an angular displacement. It can increase or decrease the total energy in a particular system.

Eqn $W = \tau\theta$

translational — KINETIC ENERGY — rotational

Name Linear kinetic energy
Symbol KE_{LIN}
Defn Kinetic energy is the energy an object has due to its motion. It is brought about by work being done to accelerate the object: $W = \Delta KE$.

Eqn $KE = {}^1/_2 mv^2$

Name Rotational kinetic energy
Symbol KE_{ROT}
Defn Kinetic energy is the energy an object has due to its motion. It is brought about by work being done to accelerate the object: $W = \Delta KE$.

Eqn $KE = {}^1/_2 I\omega^2$

Conservation of total kinetic energy: The total kinetic energy of a system is made up of its linear kinetic energy and its rotational kinetic energy: $KE_{TOT} = KE_{LIN} + KE_{ROT}$.

Therefore, kinetic energy within a system can be exchanged between linear **and** rotational types.

translational — POWER — rotational

Name Power
Symbol P
Defn Power is the *rate* at which work is done.

Eqn
$P = W/t$
$P = F \cdot d/t$
$P = Fv$

Name Power
Symbol P
Defn Power is the *rate* at which work is done.

Eqn
$P = W/t$
$P = \tau \cdot \theta/t$
$P = \tau\omega$