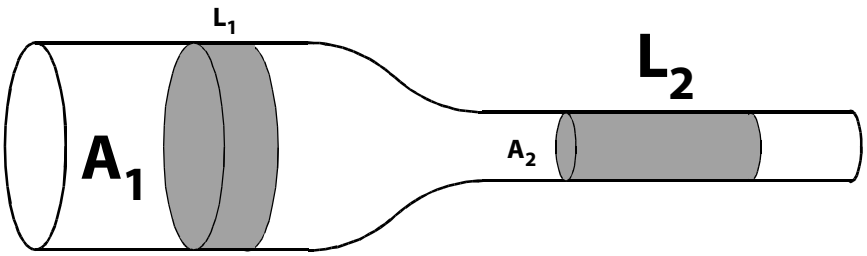
PhyzSpringboard: Continuity

THE CONSTRICTION

A pipe with a constriction carries a continuous flow of water. Consider a mass of water in the wide section. Its volume is equal to the cross-sectional area A_1 of the wide section multiplied by the length L_1 along the pipe. The constriction has a cross-sectional area of A_2 and the same volume of water will have a length L_2.



1. How do A_1, A_2, L_1, and L_2 relate?

$$A_1L_1 = A_2L_2$$

2. If that volume of water passes a point in the wide section in a time t, the same volume of water will pass a point in the constriction in a time t.

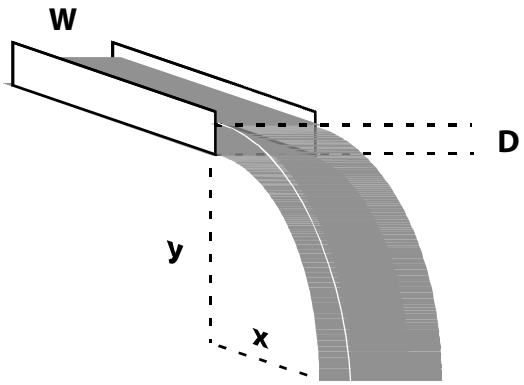
a. Express the volume flow rate (V/t) in both sections in terms of A, L, and t.

$$A_1L_1 / t = A_2L_2 / t$$

b. Rewrite those expressions taking into account that the speed of the flowing water is v = L/t.

$$A_1v_1 = A_2v_2$$

c. Box the expression above: it's the continuity equation!



A SLUICE OF LIFE

A rectangular waterway (a sluice) W = 0.50 m wide carries water at a depth D = 0.12 m.The water passes over a precipice y = 2.7 m high and lands x = 0.68 m forward of the precipice.

1.How fast is the water traveling when it crests the precipice?

Kinematics!

x: UM
$x = 0.68$ m
$v_x = ?$

y: UAM
$y = 2.7$ m
$v_{y0} = 0$
$v_y = ?$
$a = 9.8$ m/s^2

$t = ?$

$y = v_y t + 1/2\ at^2$
$y = 1/2\ at^2$
$t = \sqrt{(2y/a)}$

$v_x = x/t$
$v_x = x/\sqrt{(2y/a)}$
$v_x = 0.68$ m $/ \sqrt{(2 \cdot 2.7\ m/s\ /\ 9.8\ m/s^2)}$
$v_x = 0.92$ m/s

2.What is the volume flow rate of water in the sluice?

$V / t = A \cdot v = W \cdot D \cdot v = 0.50\ m \cdot 0.12\ m \cdot 0.92\ m/s = 0.055\ m^3/s$

3.How long would it take for this flow to fill an 8.0 m^3 storage tank?

$V / t = Av$
$t = V / Av$
$t = 8.0\ m^3 / 0.055\ m^3/s$
$t = 146\ s = 2\ min\ 26\ s$