PhyzSpringboard: Newton's Law of Cooling

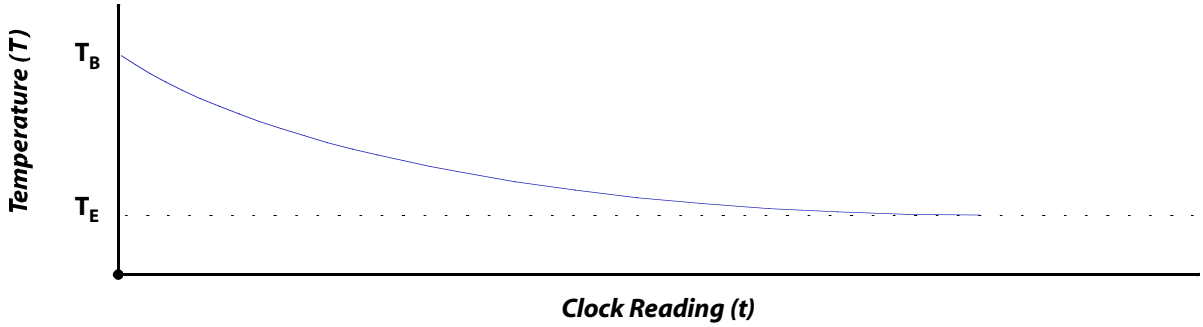
When a hot body is placed in a cooler environment, it cools down. That is, it loses internal energy to the environment. It loses internal energy by heat transfer: it conducts to cooler air around it, convection currents enhance conduction by bringing new cool air into contact with the body, and it radiates heat to the environment to a greater extent than it absorbs heat from the environment.

When taken together, the effect of all this heat loss is that the temperature of the body drops. Sir Isaac Newton is given credit for the generalization that the rate at which heat is lost is roughly proportional to the difference in temperature between the body and the environment.

1. Write Newton's Law of Cooling using symbols.

$$Q/t \propto T$$

2. Suppose a body starts out at a certain temperature, T_B, and is surrounded by an environment at a lower temperature, T_E. What would a plot of temperature versus clock reading look like?



Often, this cooling is undesirable: a plate of hot food cools down before we eat it, we go out into cold weather and cool to the point of discomfort or beyond, and so on. We have grown accustomed to certain solutions to this cooling problem: we eat our hot food quickly, we wear a jacket to insulate us from cold air.

Some cooling problems have less obvious solutions. Consider the following example of undesirable cooling and how best to deal with it.

Consider a house that is left during the day in the winter. The owner ponders what to do about the thermostat (which controls the furnace by turning it on when the temperature drops below a certain point). The owner has an interest in saving money on her energy bill and has no reason to keep the house warm while she is away.

When she leaves in the morning, should she turn the thermostat down, turn it off, leave it on all the way, or would each of these have the same result? Before we investigate this question further, record your selection and brief supporting reasoning. She should ___turn the thermostat down ___turn it off ___leave it on all the way ___each of these have the same result because...

We will now go on a seemingly unrelated excursion. The reason for this should eventually become apparent.

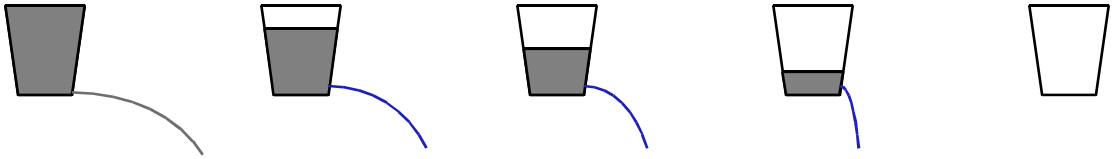
3. THE LEAKY CUP. Consider a cup with a hole near the bottom.

a. Under what conditions will the cup leak?

As long as there's water in it

b. What happens to the rate at which the cup leaks?

i. Complete the diagrams below by sketching the water stream as the cup leaks.



ii. The rate of leaking is denoted m/t and the level of water in the cup is denoted h. How—if at all—are these quantities related?

$m/t \propto h$

c. Suppose you filled the cup but weren't going to be able to drink the water until several hours passed. To be able to drink a cupful of water several hours later and waste as little water as possible, you should

___keep water running into the full cup at a rate equal to the rate of leaking,

___allow half the water to run out and keep water running into the cup at the lower rate of leaking (understanding that you'll have to refill half the cup when you eventually wish to take the drink), or

___allow all the water in the cup to leak out and turn off any water running into the cup (understanding that you'll have to refill the cup completely when you eventually wish to take the drink) because...

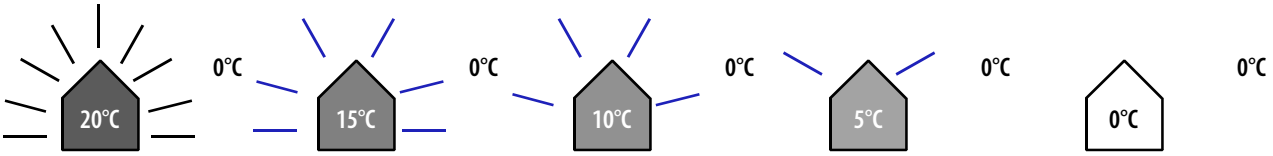
4. THE LEAKY HOUSE. Consider a house with warm air inside and surrounded by cold air.

a. Under what conditions will the house lose heat?

As long as it's warmer than its surroundings.

b. What happens to the rate at which the house loses heat?

i. Complete the diagrams below by sketching radiation emitted by the house as it cools.



ii. The rate of heat loss is denoted Q/t and the difference in temperature between air inside and outside the house is denoted T. How—if at all—are these quantities related?

$Q/t \propto T$

c. Suppose the house were warm but you were going to be away for several hours. To save energy, you should

___keep the thermostat at 20°C and keep adding heat to the house at the same rate it loses heat,

___turn the thermostat down to 10°C and add heat to the house at the lower rate at which it loses heat (understanding you will have to heat the house from 10°C to 20°C when you return), or

___turn the thermostat off and allow the house to cool completely (understanding you will have to heat the house from 0°C to 20°C when you return) because...