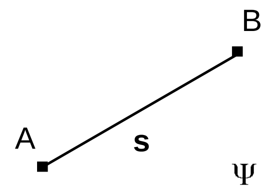
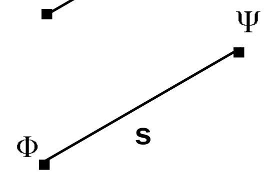
PhyzGuide: Polar Coordinates

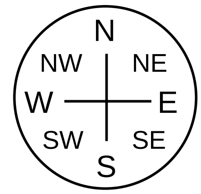
CONVENTIONAL WISDOM

Vectors convey information about magnitude and direction. So how do you express a vector? (How do you “write it out?”) A vector is most often written in one of two ways: *polar* or *rectangular* components.

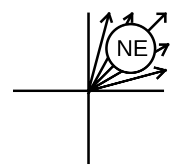


Let’s go back to that example of starting at point **A** and walking 100m to point **B**. The vector we know and love as “**s**” specifies a displacement by the distance of 100m in a particular direction. The displacement vector does not specify a starting point, an ending point, or even the path taken from **A** to **B**. Indeed, *any* displacement of 100m in the same direction as **s** *anywhere* could also be represented by **s**. (Like the displacement from Φ to Ψ.)

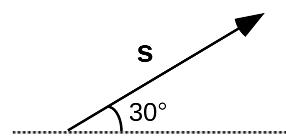




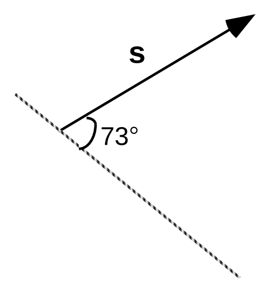
If you were asked to identify the direction of **s**, you might say, “northeast,” and you would be right. There is a convention under which up is north, right is east, down is south, and left is west (just like reading a map). The problem with calling the direction of vector **s** “northeast” is that it’s not very exact. Any direction north of east and east of north could be called “northeast.” To be more precise, you might instinctively draw in an *x*-axis and refer to the direction of **s** as an angle with respect to that axis. You would be quite correct to do that, too.



But, but, but... Keep in mind that your choice of an *x*-axis was an arbitrary decision made by you. True, we usually consider straight to the right to be the positive *x*-direction, but that is simply a convention that we have grown accustomed to. You could have just as easily (and “legally”) chosen another direction as your positive *x*-direction, as long as the *vector direction* stays the same (it may be 30° from one reference and 73° from another, but its own direction remains the same). There is no absolute coordinate system in the universe; we impose coordinates on nature to simplify our analysis of nature. *The rules of the universe work the same in all directions:* There seems to be no preferred direction in nature (well, *almost* no preferred direction, but that’s a story for another day). Nevertheless, one way of writing a vector is to give its magnitude and angle in *coordinate pair* form. **The general form of polar coordinates is**



s = (*magnitude; direction*) or, symbolically, **s** = (s; θ).

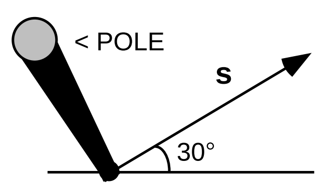


Our vector **s** would be written as

s = (100m; 30°)

since s = 100m *(notice: not 100 but 100m)*
and $\theta = 30°$.

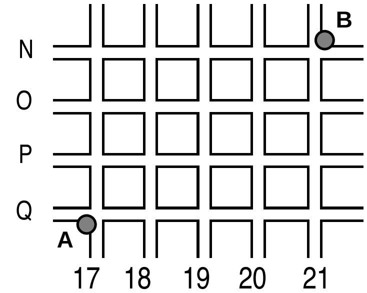
IMPORTANT: Magnitude is *always* positive (i.e. $s > 0$)! There is no such distance as –2m. Also, the direction should be given as an angle between 0° and 360° (i.e. $0° \leq \theta < 360°$).



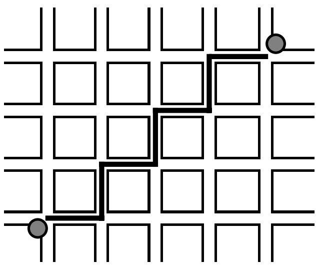
These are called **polar coordinates** not for reasons of climate or geography, but because s—the magnitude of **s**—is regarded as a length extending at an angle θ from a fixed *pole*.

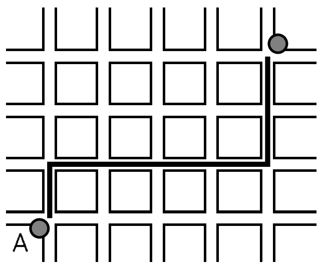
PhyzGuide: Rectangular Coordinates

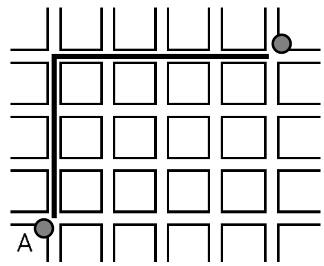
TAXI-CAB GEOMETRY

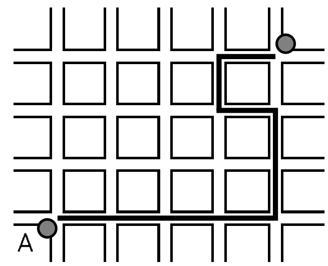


In many big cities, the downtown area is laid out like a grid with square blocks. Suppose you were in downtown Metropolis and you needed to get from the corner of 17th and Q to 21st and N in a hurry. You might flag down a taxi and tell the driver where you want to go. The cab might take any of the following routes:









Or the cab might follow a route not pictured above. Which route is best? Only the driver knows for sure. If you had said, “Take me three blocks north and four blocks east,” the driver would most likely have taken the third route shown above. The big point to notice here is that all routes produced the same exact displacement! All had the net (overall) effect of carrying you four blocks east and three blocks north.

That displacement might be described in terms of components (parts) of the total displacement. The components used to describe the displacement would be the *x*-part of the trip and the *y*-part of the trip:

$$\mathbf{x} = 4 \text{ blocks}$$
$$\mathbf{y} = 3 \text{ blocks}$$

To represent the displacement vector in terms of its *x*- and *y*-components, use *coordinate pair* form again. The general form is

$$\mathbf{s} = (s_x, s_y)$$

So for our case, we would write

$$\mathbf{s} = (4 \text{ blocks}, 3 \text{ blocks})$$

IMPORTANT: The values of s_x and s_y can be any real number (i.e. $-\infty < s_x < \infty$ and $-\infty < s_y < \infty$). [Was this also true for **s** in polar notation?]

Each component of the vector can be regarded as a vector itself. You could certainly undergo a displacement of 4 blocks east and zero blocks north (4 blocks, 0 blocks) and then undergo a displacement of zero blocks east and three blocks north (0 blocks, 3 blocks). The net effect of those two displacements is mathematically equivalent to having flown over the rooftops to make the displacement in a straight line. So we can legally write

$$\mathbf{s} = (4 \text{ blocks}, 0 \text{ blocks}) + (0 \text{ blocks}, 3 \text{ blocks}) = (4 \text{ blocks}, 3 \text{ blocks})$$

...Hmmm, could this be what they call "vector addition?"