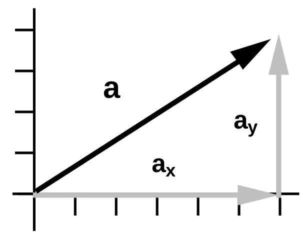
PhyzGuide: Vector Arithmetic

WHEN VECTORS COLLIDE, PART ONE: VECTOR ADDITION

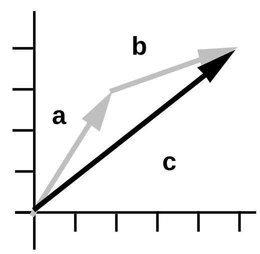
We saw earlier how it is possible to "construct" a vector from its orthogonal **components.** (Orthogonal means "at right angles," such as horizontal and vertical or *x* and *y*.) If we are given a vector **a** = (6 units, 4 units), we can think of that vector as the sum of two *component vectors*. The *x*-component $\mathbf{a_x}$ = (6 units, 0 units) and the *y*-component $\mathbf{a_y}$ = (0 units, 4 units).

$$\mathbf{a} = \mathbf{a_x} + \mathbf{a_y} = (6 \text{ units}, 0 \text{ units}) + (0 \text{ units}, 4 \text{ units}) = (6 \text{ units}, 4 \text{ units})$$

All this may be fairly obvious (if you find yourself saying things like "No Duh!" to the above, it is fairly obvious). It should also be fairly obvious that we can obtain a result *graphically* that is equivalent to the above *algebraic* statement. See the graph: to add components, we lay out the first component, then attach the tail of the next component to the head (arrow) of the first. So if we start at the origin and go 6 units to the right and 4 units up, we end up at (6 units, 4 units).



The next point on our "obvious" checklist is that we can apply these principles to actual vectors (not *just* the components). For example: suppose **a** = (2 units, 3 units) and **b** = (3 units, 1 unit). We can graph the addition as shown to the right. The **resultant vector c**, which extends from the beginning of **a** to the end of **b** is **c** = (5 units, 4 units).



Or, we can add the vectors **algebraically** by adding the *x*-components by themselves and then adding the *y*-components by themselves. Earlier, we dealt with components as component vectors: $\mathbf{a_x}$ = (6u, 0u). We can also deal with components as scalars: a_x = 6u (this is the preferred method).

$$\mathbf{c} = (a_x + b_x\,,\ a_y + b_y) \quad \text{or, in general, } \mathbf{r} = (\Sigma x\text{-components}, \Sigma y\text{-components})$$

In that case, we find **c** = (2 units + 3 units, 3 units + 1 unit) = (5 units, 4 units).

THE SMALL PRINT

Doesn't it bother you that in our graphical additions, we did not draw all the vectors from the origin (0,0)? Previously, we drew all vectors from the origin, and now we're casually "floating" vectors around so that the tail of one joins the head of another.

This "floating" quality is an important property of all vectors. Recall that when we were first introduced to vectors, we noted that a displacement vector **s** might indicate a displacement from point A to point B, but it might also indicate a displacement from point Φ to point Ψ. We also learned that the vector does not indicate a starting point, an ending point, or a path from one point to the other. It simply specifies a net movement in terms of magnitude and direction.

So we *can* slide the vector around on the plane (though we won't do it unless it serves a purpose). As long as we don't alter the vector's length (magnitude) or turn it in some new direction, *IT REMAINS THE SAME VECTOR!*

PhyzGuide: More Vector Arithmetic

THE STORY SO FAR...

Before we get into vector subtraction, let's summarize vector addition. The preferred method of vector addition is the algebraic method, because it's less cumbersome and more accurate. Again, given vectors **a** and **b** that take the form:

$$\mathbf{a} = (a_x, a_y) \qquad\qquad \mathbf{b} = (b_x, b_y)$$

(notice that the components themselves are scalars)

The *resultant vector* **c** is then figured such that

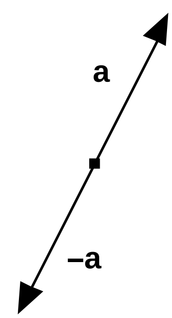
$$c_x = a_x + b_x$$
$$c_y = a_y + b_y \qquad \textit{and} \qquad \mathbf{c} = (c_x, c_y)$$

WHAT IS GIVEN CAN BE TAKEN AWAY: VECTOR OPPOSITES AND VECTOR SUBTRACTION

Remember way back when you had Algebra I and the teacher said "subtraction is the equivalent of adding the opposite." For example 7 – 4 is equivalent to 7 + (-4). Well, vector subtraction works the same way.

But wait. A **number opposite** is easy enough: you simply take the negative. The opposite of +4 is -4; the opposite of -12 is +12.

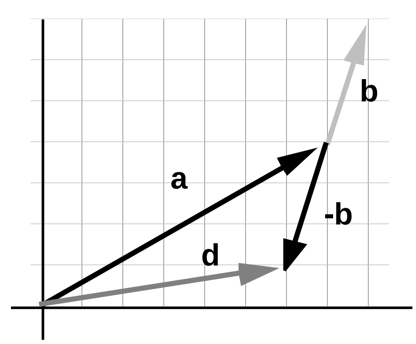
What, however, is a **vector opposite**? The opposite of a vector is a vector of *equal* magnitude in the *opposite* direction. A vector opposite is designated (as you would expect) by a negative sign. Thus the opposite of **a** is written **-a**. If **a** = (a_x, a_y) then **-a** = ($-a_x$, $-a_y$)



If vector **b** was subtracted from vector **a** to produce vector **d**, it would work like this:

$$\mathbf{d} = \mathbf{a} - \mathbf{b}$$
$$d_x = a_x - b_x$$
$$d_y = a_y - b_y \qquad \textit{and} \qquad \mathbf{d} = (d_x, d_y)$$

Graphically, we can still use the tail-to-head method in subtraction, but we must add the vector opposite of the vector we're subtracting.



OK, OK, I'll put in some numbers to "flesh it out."
a = (7 units, 4 units) **b** = (1 units, 3 units)
d_x = 7 units – 1 units = 6 units
d_y = 4 units – 3 units = 1 unit
d = (6 units, 1 unit)